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Comment on

Risk analysis in investment appraisal based on the Monte Carlo simulation technique by A. Hacura, M. Jadamus-Hacura and A. Kocot

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Abstract. Since Hertz major work on investment appraisal using the Monte Carlo Simulation technique, the so called "Risk Analysis" has become a standard tool for supporting investment decisions [1,2]. A main problem in investment appraisal is to consider and specify the risk of investment projects in an appropriate way, for enabling consistent project evaluation. In calculating a risky project's net present value (NPV) the major difficulty is to quantify the project's risk for quantifying an appropriate risk adjusted discount rate (RADR). Theoretically not founded risk adjusted discount rates face a lot of critique. Furthermore it is discussed that the incorporation of a constant risk factor into the discount rate makes a certain assumption about the resolution of uncertainty over time [3] and finally that a single net present value could not in general reflect risk properly. Especially in consequence of the last point the proponents of simulation argue that a whole distribution of net present values shows a project's risk better than a single number. In the special issue "Econophysics" of this journal Hacura *et al.* tried to describe the methodology and use of Monte Carlo Simulation in investment appraisal [4]. The purpose of this comment is to point out three fundamental flaws in that article.

PACS. 02.70.Uu Applications of Monte Carlo methods

1 How to calculate Net Present Value (NPV)

Independent of the number of steps the basic literature to risk analysis breaks down the whole process into the following stages [1,2,5]:

- Developing and building a quantitative model for the investment project considering all relevant key factors (input variables) of the expected project results (output);
- Estimating the probability distributions of the risky input variables;
- Considering stochastic dependencies (correlations) between the input variables;
- Calculating the probability distribution of the simulation output;
- Statistical analysis and interpretation of the output of simulation.

Before running a simulation it has to be specified what the output should be. Different criteria are proposed in the literature for investment appraisal, *e.g.* net present value (NPV), internal rate of return (IRR) or the payback period. Hacura *et al.* chose NPV as the appropriate simulation output, because it is the "basic decision rule for a project appraisal" [4] (p. 551). In describing this basic rule they argue that the NPV has to be calculated in "using certainty equivalent values as inputs and discounted at a rate adjusted for risk" [4] (p. 551). It will be shown that this is inappropriate.

In general future cash flows are uncertain. There are two possible ways for calculating the NPV of a risky investment: the risk adjusted discount rate (RADR-) method and the certainty equivalent (CE-) method. Both lead to the same result under certain assumptions. The RADR-method takes the riskiness of an investment project into account by adjusting the discount rate. The expected cash flows $E(\tilde{X}_t)$ are to be discounted at a RADR, which consists of the risk-free rate i plus a risk premium r_p . The higher the risk, the higher the discount rate. The present value (PV_0) formula for the simple one-period case is the following:

$$PV_0 = \frac{E(\tilde{X}_1)}{(1 + i + r_p)} \cdot \quad (1)$$

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Under the CE-method the riskiness of an investment project is not considered by adjusting the discount rate, but by adjusting the expected cash flow. The necessary adjustment can be described by the question: What is the minimum certain payoff for which one would exchange the risky cash flow? This is the certainty equivalent. The difference between the expected cash flow and the certainty equivalent is the (absolute) adjustment for risk R_A. The higher the risk, the higher the adjustment, the lower the certainty equivalent. Since the certainty equivalent is certain by definition (it is the value equivalent of a risk-free cash flow), it has to be discounted at the risk free rate

$$PV_0 = \frac{E(\tilde{X}_1) - R_A}{(1+i)} = \frac{CE(\tilde{X}_1)}{(1+i)} \cdot \tag{2}$$

The basic valuation principle behind both methods is, that by calculating net present values both numerator and denominator of the NPV formula must correspond in their dimensions of risk:

- If the numerator is a expected cash flow (*i.e.* risky) then the denominator must be adjusted for risk;
- If the numerator is a certainty-equivalent (*i.e.* riskless) then the denominator has to be riskless.

Otherwise the valuation formula for an investment project would either leave its risk unconsidered or consider its risk twice.

For a simple one-period example with one future cash flow $\tilde{X}_1$ and a (riskless) initial capital expenditure X_0 the net present value can be calculated with both methods as follows:

$$NPV_0 = \frac{CE(\tilde{X}_1)}{(1+i)} - X_0 = \frac{E(\tilde{X}_1)}{(1+i+r_p)} - X_0. \tag{3}$$

To assure that both methods lead to the same net present value the risk adjustments must correspond to the following derivations of formula (3):

$$r_p = (1+i)\left(\frac{E(\tilde{X}_1)}{CE(\tilde{X}_1)} - 1\right) \tag{3a}$$

$$CE(\tilde{X}_1) = E(\tilde{X}_1)\frac{(1+i)}{1+i+r_p} \cdot \tag{3b}$$

Of course equivalence can also be shown for the more realistic multi-period case.

For both methods the adjustment for risk may be quantified by two ways: first the individual based approach or second the capital market based approach. For the individual based approach, it is necessary to know the investor's subjective risk utility function. Then the certainty equivalent may be directly calculated from that. To derive the corresponding RADR the CE has to be transformed after formula (3a). To make capital market based adjustments neoclassical finance theory has developed the capital asset pricing model (CAPM) [6,7] (pp. 195). The output of this model is the risk premium r_p for an investment with a risky return $\tilde{r}$ (for the one period case $\tilde{r} = \frac{\tilde{X}_1}{PV_0} - 1$ and by definition $\beta = \frac{\text{cov}(\tilde{r}, \tilde{r}_M)}{\sigma_M^2}$). The expected market return is $E(\tilde{r}_M)$ and σ_M^2 is the variance of the expected market return. From the CAPM it follows the one-period present value formula with a RADR:

$$PV_0 = \frac{E(\tilde{X}_1)}{1+i+\underbrace{\beta(E(\tilde{r}_M)-i)}_{=r_p}} = \frac{E(\tilde{X}_1)}{1+i+\underbrace{\text{cov}(\tilde{r},\tilde{r}_M)\frac{E(\tilde{r}_M)-i}{\sigma_M^2}}_{=r_p}} \cdot \tag{4}$$

The corresponding present value formula with a market based certainty equivalent may be derived from formula (4):

$$PV_0 = \frac{E(\tilde{X}_1) - \overbrace{\text{cov}(\tilde{X}_1, \tilde{r}_M)\frac{E(\tilde{r}_M)-i}{\sigma_M^2}}^{=R_A}}{1+i} \cdot \tag{5}$$

Independent of how the risk adjustment is quantified (individual or capital market based), net present value has to be calculated either with expected cash flows discounted at a risk adjusted discount rate or with certainty equivalents discounted at a risk-free rate.

2 How to calculate a distribution of NPV's as output of Monte Carlo simulation

After describing how to calculate net present values of risky projects it is the purpose of this comment to discuss the appropriate discount rate to be used in calculating the net present values in a Monte Carlo simulation approach.

A lot of the early work on simulation analysis was made, before it was known how to introduce risk into calculations of net present value. For the purpose of reflecting the risk of an investment properly the time value of money was separated from risk. To do this net present values were simulated by discounting the single cash flows of each simulation run with the risk-free rate and the investment's risk was represented by drawing the probability distribution of these net present values. It was argued that this distribution of net present values shows a project's risk better than a single number does because each NPV of this distribution only considers the time value of money.

Hacura *et al.* as proponents of another school of thought believe that the most appropriate discount rate used in a simulation approach is the risk adjusted discount rate, which should include a premium for systematic (or market) risk but not for unsystematic (or project risk) [4] (p. 551). The theoretical basis for that opinion is the well known CAPM. But the use of risk adjusted discount rates in a simulation approach contains a circular argument. Simulation has been developed to gain information about a project's risk. But the use of a discount rate including a premium for risk would require the knowledge of this risk. If that risk is known, a simulation analysis is not needed anymore. Given market based data about a project's risk, simulation is unnecessary. NPV can then simply be calculated by discounting expected cash flows at the RADR – *e.g.* derived from the CAPM. Furthermore simulation output cannot deliver appropriate information about risk if a risk adjusted discount rate is used as input, because it predetermines the output of the simulation model. Interpreting such a simulation output for risk analysis would be nothing else than an improper double-counting of risk [5] (p. 275).

Considering capital market theory, there is another problem with risk analysis. A simulation approach, albeit what the discount rate is of, could only deliver information about the project's specific risk (unless simulation analysis is made in the context of a portfolio of projects). However it is known that the individual (unsystematic) risk of a project is just that part of the total (systematic and unsystematic) risk which is not relevant for valuation from an investor's point of view. A perfectly diversified investor can eliminate unsystematic risk completely. Thus, in a capital market context risk analysis cannot deliver necessary information about project risk to quantify a RADR.

Here, as an opposite school of thought it will be reasoned that the most appropriate discount rate to use in a simulation approach is the risk-free rate. The first argument is, that only the risk-free rate avoids prejudging risk. A theoretical reason for discounting with the risk-free rate in simulation could be the following: each simulation run represents one possible combination of key factors *e.g.* with one resulting cash flow in period t. The question is: what is the present value of this cash flow, given that it is that cash flow which will occur? That means, in that moment when the output value of a simulation run is known, uncertainty about this value is resolved completely, *i.e.* there is no more risk. So, it has to be discounted at the risk-free rate. But the individual risk of a project is reflected only in the probability distribution of its net present values, not in a single one of these.

3 How to use simulation output for decision making

The crucial step in decision making with Monte Carlo Simulation is to translate the distribution of net present values into measures managers could use for this purpose. Hacura *et al.* suggest some of these measures, *e.g.* expected value, variance (or standard deviation), skewness, kurtosis and many more [4] (p. 553). But they do not explain what should be done with them. For example, how should a decision be made if it is known from the cumulative distribution of a project that the probability of a positive NPV is 60 percent? It is still ambiguous, whether the project is profitable or not. And by comparing it with another project with a probability of *e.g.* 50 percent it is not clear which project is better. As long as it is not known, whether projects are profitable or not it is impossible to rank them. Furthermore, such interpretation of NPV distributions contrasts its purpose. From a theoretical point of view net present value is a unique value. It states the increase of present wealth by (or in terms of present value the today's price of) an expected future payment. Another example: how should be decided between two mutual exclusive projects A and B, when both expected value and standard deviation of A's NPV distribution are greater than those of project B. To make a decision it is necessary to trade off return (expected value) and risk (standard deviation). But that is what simulation analysis cannot achieve, since neither an investor's subjective risk utility function nor a market based risk valuation is considered. This lack grows when further moments of distribution should be considered. Risk analysis is only a tool for the preparation of a decision and not a decision rule. Richard Brealey and Stewart Myers sum up that "managers can only be told to stare at the distribution until inspiration dawns. No one can tell them how to decide or what to do, if inspiration never dawns" [5] (p. 275). Finally, the question is left unanswered, what could be the economic relevance of such measures of distribution? Any parameter of the simulation output (*e.g.* higher moments of distribution like skewness or kurtosis) depends on the quality of input factors which are on their part only estimations. But what could be the insight of an output, when its quality depends mainly on the quality of estimated input data? One must be aware of a pseudo-exactitude.

4 Conclusion

This comment points out three flaws in a recent article of Hacura *et al.* First it was explained how NPV has to be calculated correctly, second the discount rate to be used in a simulation approach was discussed and third it was described that risk analysis is not a tool to make an investment decision but rather for the preparation in the preliminary stages of such a decision. Thus, risk analysis can be helpful, since it can support the projection of cash flow determinants, which can lead to a better understanding of the project and improve the insights of project risk. However, net present value should still be calculated by discounting either expected cash flows with a risk-adjusted discount rate or certainty equivalents with the risk-free rate.

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References

1. D.B. Hertz, Harvard Bus. Rev. **42**, 95 (1964)
2. D.B. Hertz, Harvard Bus. Rev. **46**, 96 (1968)
3. A.A. Robichek, S.C. Myers, J. Finance **21**, 727 (1966)
4. A. Hacura, M. Jadamus-Hacura, A. Kocot, Eur. Phys. J. B **20**, 551 (2001)
5. R.A. Brealey, S.C. Myers, *Principles of Corporate Finance*, 6th edn. (McGraw Hill, 2000)
6. W.F. Sharpe, J. Finance **19**, 425 (1964)
7. T.E. Copeland, J.F. Weston, *Financial Theory and Corporate Policy*, 3rd edn. (Addison-Wesley 1988)